Section 14.3

Motion

If we track the motion of an object in $\mathbb{R}^3$ at time t we arrive at a function $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$.

The derivative function $\vec{v}(t) = \vec{r}\,'(t) = \langle x'(t), y'(t), z'(t) \rangle$, which from Section 14.2 gives the tangent vector, will also measure the **instantaneous velocity** of the moving object.

The **speed** is given by $|\vec{v}(t)|$. The **acceleration** by $\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t)$
Compare the trajectories and velocities

$$\vec{r}(t) = \langle t, t^2 - 4, \frac{t^3}{4} - 8 \rangle$$
$$\vec{R}(t) = \langle t^2, t^4 - 4, \frac{t^6}{4} - 8 \rangle$$

Special Motions

Analyze the motion of $\vec{r}(t) = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$

Suppose $\vec{r}(t) \cdot \vec{v}(t) = 0$ for all t. Describe $\vec{r}$.

Show that $\vec{r}(t) = \langle 3\cos t, 5\sin t, 4\cos t\rangle$ lies on a sphere, find its radius.

Motion in Gravitational Fields

For an object moving in a plane, we assume a coordinate system with y-axis pointing vertically upward, and an x-axis pointing in the direction of horizontal motion. The force of gravity on the object is given by $\vec{F}_g = \langle 0, -mg\rangle$. Find the position of the object as a function of time, ignoring all forces other than $\vec{F}_g$.